

The following relate to questions asked during the August 24-25, 2010 seminar sessions.

Suppose you have several alternative investments. How do you determine the appropriate fraction of your wealth to invest?

The examples I showed in file 'kelly geomean.xls' had two related assumptions: (a) The alternative investments were mutually exclusive, (b) Uninvested capital yielded a "risk free" rate, assumed to be zero. Let's relax assumption (a)

A lot depends on the correlation among the investments. For example, suppose you have 5 investments, each with the same probability distribution, and all 100% correlated. You've determined, using some version of the geometric mean or Kelly criterion, that for such a probability distribution 50% of your "wealth" is the appropriate amount to invest, with periodic rebalancing. Since we've specified that these 5 investments are 100% correlated, it really doesn't matter how you divide that 50% among the 5 investments. You could put 10% into each of the 5 investments or 50% into just one. In the real world, it is impossible for us to have that degree of certainty regarding the correlation among the investments, so most people would lean to splitting the 50% among more than one of the alternatives.

Suppose these 5 investments were totally uncorrelated. In that case, by symmetry, the default position would be to split the 50% five ways, resulting in 10% for each investment.

Now, consider another extreme, where you have two alternative investments, each a perfect hedge for the other. They have a correlation of -1 . In that perfect world, you would be justified in borrowing as much as possible in order to invest it. Of course, there are no perfect hedges in the real world, which is often forgotten. There is always the possibility of "unknown Unknowns". Even if there were perfect long-term hedges, in the short term you can go broke, as Long Term Capital discovered.

Between these extremes, the mathematics gets unintelligible for all but professional mathematicians and finance quants (see sections 7 and 8 of <http://www.edwardthorp.com/sitebuildercontent/sitebuilderfiles/KellyCriterion2007.pdf>). A capital investment budget for a company or division may have projects with a fairly high degree of correlation, based on the economics of the industry. The R&D budget for a large pharmaceutical company may be much less correlated, especially if divided among several classes of drugs.

That is why my real world suggestions are: (a) Individual projects should be ranked based upon their semi-variance, geometric mean, Kelly #, or something similar. No one measurement should be looked at in isolation, but you do need something that considers evaluates the risk of ruin., (b) Unless you are dealing with a hedge scenario, you should think twice about investing more than $\frac{1}{4}$ of the Kelly # in a particular project.

How do we estimate probabilities for an investment or project?

I don't have a silver bullet for this issue. Frank Knight, in his book Risk, Uncertainty, and Profit observed that "Business decisions, for example, deal with situations that are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guidance." However, there are some broad generalizations that can be made.

The fact that an event is rare doesn't necessarily imply that we can't predict it with a great deal of certainty. Eclipses are an example. In general, it is easier to develop models for physical processes than human processes. In fact, when a physical model fails in the slightest detail, the failure may eventually result in a totally new paradigm. This happened in physics one hundred years ago and resulted in quantum physics and special relativity. It may well be that the failure is a result of operating in a different regime, the very small (quantum physics) or very fast (relativity).

Analogues in operations might be repeated failures in a previously smoothly running machine or a sudden increase in defects. In investing, the possibility of a regime change is one of the justifications for "stop-loss" orders. When a strategy developed based on statistical relationships no longer works, it is possible that we are now operating in a regime significantly different from the regime that generated the statistics.

A resource that I should have mentioned in the seminar is Table 3 ("Uncertainty Checklist") of a recent paper by Lo and Mueller, WARNING: Physics Envy May Be Hazardous to Your Health

http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1563882

Their taxonomy of Uncertainty in that paper is also useful.

Warren Buffett's description of how he evaluated the uncertainties in buying Wells Fargo twenty years ago is useful (<http://www.berkshirehathaway.com/letters/1990.html>)

Of course, ownership of a bank - or about any other business - is far from riskless. California banks face the specific risk of a major earthquake, which might wreak enough havoc on borrowers to in turn destroy the banks lending to them. A second risk is systemic - the possibility of a business contraction or financial panic so severe that it would endanger almost every highly-leveraged institution, no matter how intelligently run. Finally, the market's major fear of the moment is that West Coast real estate values will tumble because of overbuilding and deliver huge losses to banks that have financed the expansion. Because it is a leading real estate lender, Wells Fargo is thought to be particularly vulnerable.

None of these eventualities can be ruled out. The probability of the first two occurring, however, is low and even a meaningful drop in real estate values is unlikely to cause major problems for well-managed institutions. Consider some mathematics: Wells Fargo currently earns well over \$1 billion pre-tax annually after expensing more than \$300 million for loan losses. If 10% of all \$48 billion

of the bank's loans - not just its real estate loans - were hit by problems in 1991, and these produced losses (including foregone interest) averaging 30% of principal, the company would roughly break even.

A year like that - which we consider only a low-level possibility, not a likelihood - would not distress us. In fact, at Berkshire we would love to acquire businesses or invest in capital projects that produced no return for a year, but that could then be expected to earn 20% on growing equity. Nevertheless, fears of a California real estate disaster similar to that experienced in New England caused the price of Wells Fargo stock to fall almost 50% within a few months during 1990. Even though we had bought some shares at the prices prevailing before the fall, we welcomed the decline because it allowed us to pick up many more shares at the new, panic prices.

Investors who expect to be ongoing buyers of investments throughout their lifetimes should adopt a similar attitude toward market fluctuations; instead many illogically become euphoric when stock prices rise and unhappy when they fall. They show no such confusion in their reaction to food prices: Knowing they are forever going to be buyers of food, they welcome falling prices and deplore price increases. (It's the seller of food who doesn't like declining prices.) Similarly, at the Buffalo News we would cheer lower prices for newsprint - even though it would mean marking down the value of the large inventory of newsprint we always keep on hand - because we know we are going to be perpetually buying the product.

This is an unsatisfying answer, I know. The ultimate protection against poor estimation is to acknowledge our limitations and constrain the risk of our ignorance resulting in ruin. Thus we need to always build a reasonable probability of something/anything pretty bad happening. Clearly, we can't go too far. After all, if the sun goes nova, that would be so bad, that we wouldn't have to worry any longer about our projects or investments!

Once we've derived the Expected Value of the Logarithms (as shown in "Kelly Geomean.xls", what is the compounded rate of return?

I've developed a spreadsheet "Compound Growth Rate from Kelly.xls" which takes portions of "Kelly geomean.xls" and "Overbetting Can Lead to Ruin.xls". All three are available at www.brigadoonbay.com/5 I then modified the spreadsheet for the hedge fund example I presented at the seminar (80% chance of doubling money in a year; 20% chance of going bust; 60% of wealth invested). The left part of this new spreadsheet shows the Expected Value of the Logs to be 0.188. Since this is derived from logarithms it makes sense that we need to use it as an exponent (reversing the logarithm). So we calculate $e^{0.188}$ and then subtract 1 to get the compound rate of return (20.41%). To doublecheck the result, I compared it to the actual calculations (right side of spreadsheet)

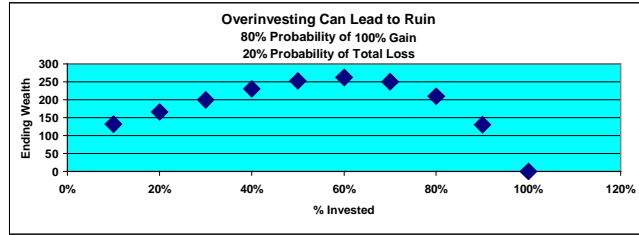
which shows an ending value after 5 years of 253, a 253% return in 5 years. That equals 20.41% annually also, which gives confidence in the correctness of the equation.

Adapted from file 'kelly geomean.xls' sheet 'Scenario 1'

percentage of wealth invested=> 50.00

Probability	Gain / (Loss)	Weighted Value of %Wealth Increase	Weighted Value of Log of %Wealth Increase
80.0%	100%	80%	0.324
20.0%	-100%	-20%	(0.139)
		0%	-
		0%	-
		0%	-
		0%	-
100%	Expected Val	60%	0.186
	Std Dev	80%	

Annualized Return=> 20.41%



Example for 50 % Invested with Rebalancing

Period	Wealth	% Invested	\$ Invested	%Gain/Loss
1	100	50%	50.00	100%
2	150	50%	75.00	100%
3	225	50%	112.50	100%
4	338	50%	168.75	100%
5	506	50%	253.13	-100%
6	253			

Annualized Return=> 20.41%

Taking the Expected Value of the Logarithms and using it as the exponent in function EXP results in the compound This sheet compares the results (in red) using two different methods for a simple example

from file: 'Compound Growth Rate from Kelly.xls'

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